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Faculty of Economics

**Risk-Price Sensitivity: An Empirical Case in the Mexican Bond
Market**

Thesis to obtain the Bachelor's Degree in Economics

Presented by:

Miguel Angel Franco Reyes

Thesis advisor:

PhD. Alfonso Mendoza Velazquez

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I Introduction

Most individuals borrow money from bank, but corporations can also borrow money from another type of financial intermediaries, like the bond and capital market.

A bond is essentially a document with a promise of future payment. Unlike stocks, bonds do not grant property rights to the company. Bond holders are just lending money to the company in exchange of a promise of future payment in a particular point in time. In addition, some bonds pay regular installments called coupon payments according to a prior date's schedule, these are known as coupon bonds.

By issuing bonds, companies can raise capital to make bigger investments and then repay their debt. But companies are not the only institutions that raise money from the bond market; federal and local governments can make use of this financial instruments as well. The investors must remember that an investment in a bond carries credit risk and interest rate risk. An upward movement in interest rates will affect the value of the bond in a negative way. On the other hand, a decrease in the interest rate will cause the bond to increase in value.

Globally the financial bond markets worth around about US\$93 trillion¹. Emerging financial markets account for about 17 percent² of the worldwide market and are growing fast, particularly in China and India. Asia and Latin America are the largest emerging financial markets.

The bond markets in Latin America are again growing in the wake of the subprime crisis in the United States in 2008. One of the markets that has been better at recovering is the Mexican market.

The bond market in Mexico has evolved very rapidly in the last few years, particularly the government securities. By the end of 2016, 64.8% of the total trades in the debt market corresponded to government securities. We are talking about a market that moves around 100 billion MXN daily (5,300 billion USD)³.

The Mexican bond market is valued in 6.2 billion MXN, mostly concentrated in government securities⁴. The government securities are traded in auctions, with banks and brokerage firms being the principal participants. The principal government bond holders are pension funds and investment companies that hold around a 30% of all bonds, while banks only hold 3.9% from all the market⁵.

The Mexican Central Bank (Banxico) it's the responsible to determine the target for the Interbank Interest Rate with which the bonds are traded. Since the end of 2015 and due market conditions such as loss of dynamism in emerging economies, the slow growth of the euro zone and Japan⁶,

¹ The World Bank (2010) What is the Context Of Bonds In Global Financial Markets?

² The World Bank (2010) What is the Context Of Bonds In Global Financial Markets?

³ Rojas, E., Zepeda, C., González, A. (2017) Esto es lo que sabemos de la investigación en el mercado de bonos. El Financiero

⁴ Cota, I., Martín, E., Davis, M. (2017) Habrá más 'ojos' en investigación de colusión en mercado de bonos. El Financiero

⁵ Rojas, E., Zepeda, C., González, A. (2017) Esto es lo que sabemos de la investigación en el mercado de bonos. El Financiero

⁶ Banco de México (2015) "Anuncio de Política Monetaria 17-12-2015"

the scenario of great uncertainty due to the fiscal, commercial and migration policies that the new Administration in the United States carried and their impact in the volatility of the financial markets⁷, led Banxico to rise the interest rate to levels of crisis not seen since mid-2009.

These increases were in line with the monetary policy of the FED, which has recently announced three more increases for the remainder of 2017.

These hikes on interest rates will rise the financial debt cost of the bonds issued by local governments and will put some pressure on their liquidity. Fitch Ratings recently announced that if the states do not restructure their debts, the hike may affect their credit rating⁸. As mentioned above, in the Mexican bond market the main holders are pension funds as well as investment companies, and their positions are concentrated mainly in government securities.

The exposure to interest rate movements in a context of contractionary monetary policy by the central banks, makes necessary for asset managers to know the amount of risk they are or will be facing in the scenario of more future interest rate hikes.

Given the market conditions and the composition of the bond market, this paper will focus in measuring the risk-price sensitivity of Mexican government bonds.

In order to analyze the case this paper will divide as following: the section II will describe the existent literature regarding risk price sensitivity, included the most know methods to measure it. In section III we will display the descriptive statistics from our data. The section IV will contain the methodology that will be applied in the analysis. Section V will explain the obtained results and finally section VI presents the conclusions and outlooks.

II Literature Review

The literature due this topic is vast enough to have a benchmark for this paper. Authors like Babbel, Merrill, and Panning (BMP) (1997) focuses in risk management measures. They compared interest sensitivities of risk-prone and risk-less bonds using option-pricing technology to calculate risk-adjusted duration and compared it to the duration of matched, synthetic Treasury bonds. They found that risk-adjusted duration was lower than riskless duration for all levels of yield spreads, with the difference monotonically increasing with yield spreads.

Other authors like Borovicka, Hansen & Hendricks (2010) constructed shock-exposure elasticities which capture the sensitivity of expected cash flows from different assets. They find that the risk premium on the maximal growth portfolio reveals information on the volatility of the logarithm of the stochastic discount factor.

In the same path Alvarez & Koskela (2006) analyzed the investment under interest rate variability conditions. Their results concluded that the interest rate variability has a decelerating or accelerating impact on investment depending on whether the current interest rate is below or above the long-run steady-state interest rate, and numerical calculations show that its quantitative size may be very large. Cornell (1999) studies the common hypothesis that longer-term projects at higher rates on the grounds are riskier. His results suggest that the survival of the

⁷ Banco de México (2016) "Anuncio de Política Monetaria 15-12-2016"

⁸ Flores, Z. (2017) "Fitch ve riesgos en la deuda de los estados por mayores tasas" *El Financiero*

corporate practice of discounting longer-term projects at higher rates is not irrational but is an intuitive response to correctly perceived risks.

Finally, Escribano (2003) mentioned on his work changes in the interest rate affect the asset prices of risky fixed income and consequently, the duration of those titles. Then we can find a correlation between the duration and risky bonds.

On his work Escribano (2003) examine the price sensitivity in the Spanish Bond Market regarding the variations of the risk free interest rates through the concept of risky duration and convexity, concluding with a model whose coefficients are able to measure the relationship between interest rate and prices in the market of fixed income. Saying that our analysis will replicate the work of Escribano (2003) and apply it in the conditions of the Mexican Bond Market.

III Descriptive statistics

Data

Our data base was constructed employing the daily logarithmic returns of 27 bonds issued by Mexican federal and local governments, by the institute for protection and bank savings (IPAB for its acronym in Spanish) and state productive enterprises. Due to convergence problems the analysis was centered in only 21 bonds. The period of analysis is from 2006 to 2016 established by the different maturities of the bonds. All the data were obtained in the platform of Thomson Reuters. The final data were subjected to a filter process, applying the Ljung-Box test for autocorrelation and the ARCH LM-test for heteroskedasticity. The selected bonds are summarized in Table 1.

Table 1

Bonds Selected				
Symbol	Issuer	Issuance date	Maturity date	Credit Rating*
D2_PEMEE62_170815	Petróleos Mexicanos	28/10/1999	15/08/2017	A+
VRZCB06U	Estado Libre y Soberano De Veracruz Llave	30/11/2006	31/07/2036	AA+
IFHGO07	Instituto Para El Financiamiento Del Estado De Hidalgo	09/05/2007	09/05/2019	A
CHIACB07U	Gobierno Del Estado De Chiapas	28/06/2007	25/06/2037	AAA
GDFCB07	Gobierno Del Distrito Federal	17/12/2007	22/11/2027	AAA
VRZCB08U	Estado Libre y Soberano De Veracruz Llave	16/12/2008	31/07/2036	AA+
D2_PEMEX4_010318	Petróleos Mexicanos	19/01/2009	01/03/2018	Baa3
D2_PEMEX_061117	Petróleos Mexicanos	18/08/2009	06/11/2017	Baa3
IS171005	Instituto Para La Protección Al Ahorro Bancario	14/10/2010	05/10/2017	A-
IQ170629	Instituto Para La Protección Al Ahorro Bancario	05/07/2012	29/06/2017	BBB+
VERACB12-2	Estado Libre Y Soberano De Veracruz Llave	12/11/2012	01/09/2027	A+
IQ171109	Instituto Para La Protección Al Ahorro Bancario	15/11/2012	09/11/2017	A
PEMEX12	Petróleos Mexicanos	29/11/2012	23/11/2017	Baa3
OAXCB07U	Estado De Oaxaca	11/12/2012	31/05/2037	AA+
CFE13	Comisión Federal De Electricidad	28/06/2013	22/06/2018	Baa1
IM170727	Instituto Para La Protección Al Ahorro Bancario	31/07/2014	27/07/2017	A
FNCOT14	Instituto Del Fondo Nacional Para El Consumo De Los Trabajadores	11/09/2014	07/09/2017	AAA
IM171123	Instituto Para La Protección Al Ahorro Bancario	27/11/2014	23/11/2017	A3
CFE15	Comisión Federal De Electricidad	09/06/2015	02/06/2020	Baa1
FNCOT16	Instituto Del Fondo Nacional Para El Consumo De Los Trabajadores	08/04/2016	05/04/2019	AAA
FNCOT162	Instituto Del Fondo Nacional Para El Consumo De Los Trabajadores	23/06/2016	21/06/2018	AAA

*The credit ratings were issued by Moody's and Fitch Ratings.

Source: Thomson Reuters

The table 2 presents the descriptive statistics from the logarithmic returns of each bond. As we can see in the table most of the returns have a mean negative value which is logical due the recent interest rates hikes by the central bank. The bonds with a longer time to maturity have the worst mean return, like the VRZCB06U that have a maturity date on 2036. Even with the market conditions, the standard deviations from the bonds seems not be so high, but this is logical due the periodicity of the data.

Table 2

Log Returns Descriptive Statistics				
Symbol	μ	σ	Kurtosis	Skewness
D2_PEMEE62_170815	-0.0001	0.0013	49.0647	5.5198
VRZCB06U	-0.0004	0.0066	7.3929	-0.4714
IFHGO07	-0.0002	0.0140	227.6738	-0.8081
CHACB07U	-0.0002	0.0118	23.1710	-0.3511
GDFCB07	-0.0001	0.0081	17.6899	-2.1666
VRZCB08U	-0.0002	0.0028	29.5425	-0.1069
D2_PEMEX4_010318	-6.4E-05	2.9E-03	1.4E+01	-9.7E-01
D2_PEMEX_061117	-8.4E-05	1.9E-03	4.6E+01	-2.5E+00
IS171005	4.5E-05	3.5E-03	9.3E+01	-4.3E+00
IQ170629	6.0E-06	6.7E-05	2.3E+00	-5.0E-01
VERACB12-2	-0.0001	0.0017	56.1399	-4.3767
IQ171109	1.5E-05	7.6E-05	4.8E+00	-5.6E-01
PEMEX12	-8.6E-06	1.6E-03	1.4E+01	-6.9E-01
OAXCB07U	-0.0005	0.0106	58.8762	-2.7502
CFE13	-2.4E-05	7.6E-04	2.7E+01	-4.9E-01
IM170727	7.6E-06	4.5E-05	1.4E+01	1.2E+00
FNCOT14	-7.0E-07	4.3E-05	7.6E+01	-4.4E+00
IM171123	6.8E-06	3.1E-05	3.6E+00	9.0E-01
CFE15	-4.9E-05	3.6E-03	3.4E+01	3.0E-02
FNCOT16	9.8E-06	1.3E-03	2.9E+01	-5.4E-01
FNCOT162	-4.4E-18	5.4E-05	1.6E+01	-1.2E+00

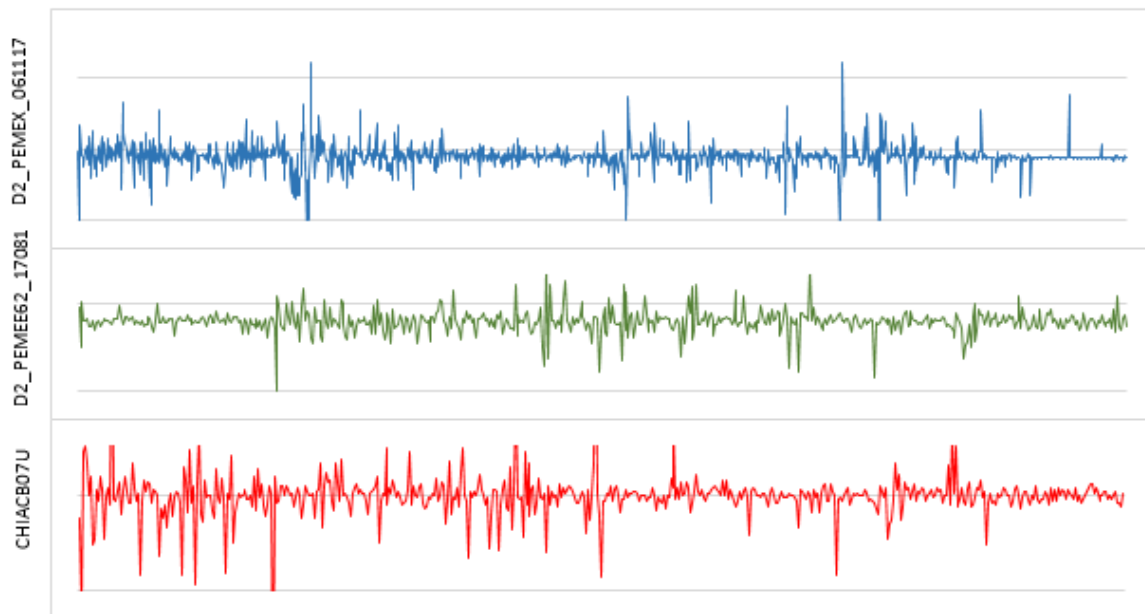
Source: own calculations

In the case of the kurtosis, most of the bonds seems to be platykurtic, with the exceptions of the bonds with higher time to maturity that the returns seems to have leptokurtic shape. The skewness suggest a negative asymmetry for almost all of the bonds.

The picture 1 show us the daily returns from three representative bonds. As we can see the behavior of the time series appears to be consistent with the volatility movements in groups that suggest a model a GARCH models later.

Picture 1

Daily Log Returns

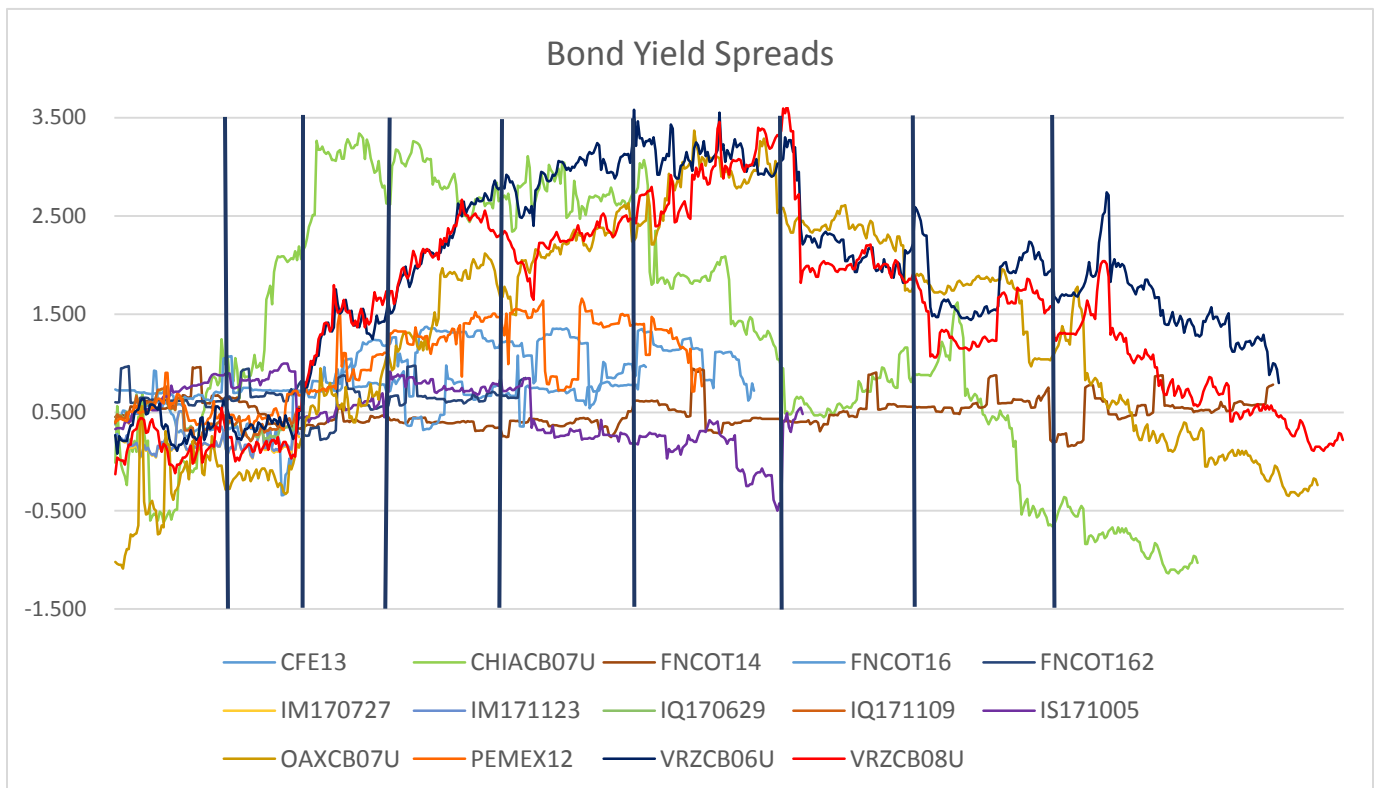


Source: Thomson Reuters

Our data also show us the actual effects from the monetary policies in the bonds. The picture 2 show us the yield spread of most of the bonds⁹ since the end of 2015, when the interest rate hikes began. The blue vertical lines displayed in the graph represent an interest rate change from the Mexican Central Bank between November 2015 and March 2017. This graph allow us to understand the actual effect that the changes in interest rates carries, with spreads that exceed the 300 bsp.

⁹ Some bonds with outliers' values were excluded from the graph due presentation reasons.

Picture 2



Source: Banco de Mexico and Thomson Reuters

IV Methodology

Duration

McCauly (1938) introduced the concept of duration as a measure for sensitivity of the bonds price due a changes on interest rates. Essentially reveals how long it will take for the interest payments generated by a fix income investment to repay the investment principal. It is expressed as a number of years. A higher duration number means that the bond price is more sensitive to interest rate changes while a lower duration number means a bond's price is less sensitive to interest rate changes.

The duration is a complicated calculation but the needed information it's provided in every trade platform. In order to calculate the McCauly Duration the California Debt and Investment Advisory Commission uses the following formula:

$$\text{Macaulay Duration} = \sum_{t=1}^n \frac{(\text{PV})(\text{CF}_t) \times t}{\text{Market Price of Bond}}$$

Definitions:

$(\text{PV})(\text{CF}_t)$ = present value of coupon at period t

t = time to each cash flow (in years)

n = number of periods to maturity

Source: California Debt and Investment Advisory Commission

To measure the sensitivity of changes in interest rates Escibano (2001) propose a model based on the concept of risky duration defined by Fisher (1966) and Hopewell and Kaufman (1973):

$$\frac{dP_t^k}{P_t^k} \cong \frac{D_t^k}{(1 + y_t^k)} (dy_t^k) \quad (1)$$

Where:

- P_t^k = asset price in t
- y_t^k = yield to maturity in t
- D_t^k = risky (or modified) Duration

To obtain the Modified Duration we applied the calculation proposed by Dixit & Pindyck (1994) as:

$$D_t = \frac{\text{Duration}}{(1 + r_t)} \quad (2)$$

Its important to rememeber that the price of the assets (in our case the bonds) is a function of the yield to maturity (y_t^k) and the yield spread (for now on r_t). Following this we have a yield to maturity denoted by $y_t = i_t + r_t$, therefore we can moddify equation 1 as:

$$dP_t = \frac{\partial P_t}{\partial i_t} di_t + \frac{\partial P_t}{\partial r_t} \frac{dr_t}{di_t} di_t \quad (3)$$

Plugging (3) into equation (1) we have:

$$\frac{dP_t}{P_t} = -\frac{D_t}{1 + y_t} di_t - \frac{D_t}{1 + y_t} \frac{dr_t}{di_t} di_t$$

(4)

So

$$\frac{dP_t}{P_t} = -\frac{D_t}{1+y_t} \left(1 + \frac{dr_t}{di_t}\right) di_t$$

(5)

$$\frac{dP_t}{P_t} = -\frac{D_t}{1+y_t} \alpha di_t$$

(6)

The table 3 summarize the effects of α .

Table 3

Coefficient α effects

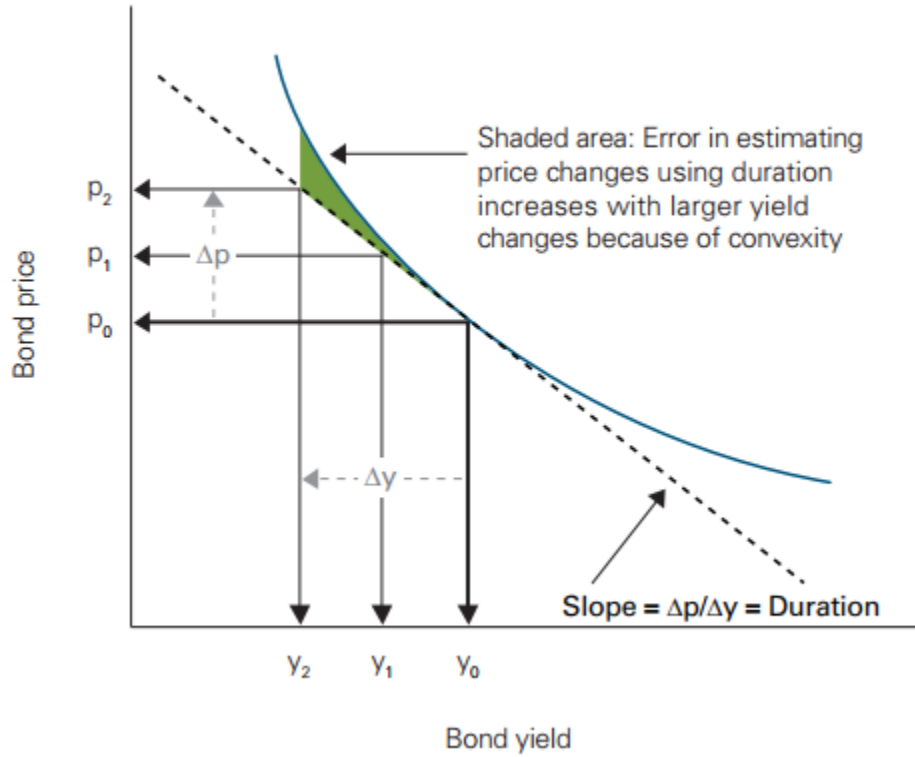
α	Effect
If $\alpha > 1$	An increase in the interest rate implies increment in the yield spread
If $\alpha = 1$	There is independence between interest rates changes and yield spread
If $\alpha < 1$	There is an inversal effect between interest rate changes and yield spread

Source: Escibano (2003)

Convexity

Duration only captures a lineal aspect of the relationship between bond price and interest rate changes. For larger interest rates moves the relationship between the changes in bond prices turns to be asymmetric. Bennyhof & Zilbering (2010) mention that the bond price decrease resulting from a large interest rate increase will generally be smaller than the price increase resulting from an interest rate decline of the same magnitude. This asymmetry arises from the convex payoff pattern shown in the relationship between bond prices and bond yields. Picture 3 show the graphical differences between Duration and Convexity

Picture 3



The convex shape shows that a portion of the interest rate move remains uncaptured by duration.

Source: Bennyhof & Zilbering (2010)

Continue with the work of Escribano, Later on proceed to the introduction of the concept of Convexity to the model. The introduction of this second factor goes in the same line with the work of Ilmanen, McGuire and Warga (1994) where they mention that this factor should contribute to the marginal increment of the determination coefficient.

To calculate the Convexity of the bonds we applied the calculation proposed by Dixit & Pindyck (1994) as:

$$C = \frac{\Delta Price}{\Delta Return^2} \quad (6)$$

Now the two factors model will be expressed in the following equation:

$$\frac{dP_t}{P_t} = \alpha \frac{D_t}{1 + y_t} di_t + C \frac{1}{2} \left[\frac{d^2 P_t}{dy_t^2} \frac{1}{P_t} \right] (dy_t)^2 + \varepsilon_t \quad (7)$$

Ilmanen, McGuire and Warga (1994) mention that the addition of a second factor should contribute to the marginal increment of the determination coefficient.

GARCH & EGARCH Models

The models proposed before were linear and calculated with the OLS methodology, under the hypothesis of constant variance across the time. However most of the financial variables show different volatility across the time and consequently a changing variance. In order to solve this issue it is useful to make use of a GARCH model. Escribano (2003) made use of the GARCH model proposed by Duffee (1999).

Duffee (1999) assume a process GARCH (1 1)¹⁰, this explained by the behavior of the yield from the Mexican market. The process is structured in two equations, one with the return of the bond as a conditional mean and another one for the conditional variance the equation (8) applied the one Duration one factor model and the equation (9) includes the Convexity:

$$R_t = \frac{D_t}{1 + y_t} \alpha_t di_t + \varepsilon_t$$
$$Var_{t-1}(\varepsilon_t) = \theta_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta_1 \theta_{t-1}^2$$
(8)

$$R_t = \frac{D_t}{1 + y_t} \alpha_t di_t + c_t \frac{1}{2} \left[\frac{d^2 P_t}{dy_t^2} \frac{1}{P_t} \right] (dy_t)^2 + \varepsilon_t$$
$$Var_{t-1}(\varepsilon_t) = \theta_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta_1 \theta_{t-1}^2$$
(9)

In order to deal with the asymmetric volatility in the time series Escribano make use of an asymmetric volatility model EGARCH (1 1). The model is represented with the following equation:

$$\ln \sigma_t^2 = \omega + \alpha \left(\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \left(\frac{2}{\pi} \right)^{1/2} \right) + \beta \ln \sigma_{t-1}^2 + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$
(10)

VI Results

Duration

The first step of our analysis is the regression between bond returns and their modified duration with the following equation.

$$Return_t = \beta + \alpha Modified\ Duration_t + \varepsilon$$
(11)

¹⁰ Different GARCH processes were calculated, but the GARCH (1 1) displayed the more realistic results.

With this we want to estimate the duration coefficient to have a first idea of price sensitivity of the Mexican bonds markets. According to the model proposed by Escribano (2003), the coefficient α describes the reaction of the bond to a change in the interest rate.

Table 4

Duration Model					
Symbol		Coefficient α	R2(%)	SE	Q (12)
CFE13	*	-0.000051	0.890000	0.008900	* 34.8245
CFE15		-0.000751	0.440000	0.000527	* 364.965
CHACB07U		-0.000013	0.060000	0.000021	20.1334
D2_PEMEE62_170815		-0.000016	0.090000	0.000900	4.5214
D2_PEMEX_061117		-0.000001	0.000000	0.000022	* 28.6557
D2_PEMEX4_010318	*	0.000001	0.220000	0.000000	* 76.0621
FNCOT14		0.000000	0.060000	0.000001	* 0.096
FNCOT16		0.000012	0.020000	0.000056	58.328
FNCOT162		0.000000	0.010000	0.000002	16.1947
GDFCB07		-0.000008	0.030000	0.000015	* 25.9859
IFHGO07		-0.000464	0.170000	0.000469	* 116.083
IM170727		0.000025	3.440000	0.000013	10.9358
IM171123		0.000012	5.690000	0.000005	14.0698
IQ170629		0.000028	0.910000	0.000030	13.3458
IQ171109		0.000016	4.570000	0.000007	10.2736
IS171005		0.000030	0.080000	0.000056	* 93.5263
OAXCB07U		0.000025	0.090000	0.000034	* 59.4442
PEMEX12	*	-0.000003	0.000000	0.000063	* 31.4419
VERACB12-2	*	-0.000117	0.460000	0.000070	* 37.9315
VRZCB06U		-0.000081	0.060000	0.000136	* 28.1284
VRZCB08U		-0.000057	0.200000	0.000050	* 26.3017

Note: This table shows the results of the OLS regression of the model presented in the equation (6). The coefficient α measures the sensitivity among the bonds and the risk free interest rate. R2 represents the coefficient of determination and Q(12) is the statistical of autocorrelation proposed by Ljung-Box of 12 order. * denotes the significance at the 5 percent level

Source: own calculations

First, we verify that all coefficients have a negative sign as we expected, however the bonds with the shorter time to maturity (between 3 and 9 years to maturity) have a positive sign. This suggest that an increase in the interest rate implies increment in the yield spread, so a long position in this shorter bonds would be strongly recommended.

On the other hand, most of the results belongs to the bonds with a longer time to maturity. Those bonds presents a negative sign, which explain us that there is an inverse effect between interest rate changes and yield spread.

In most of the cases we cannot say that our results are significantly different from 0. The results provided by the R square suggest us that there is not a significant linear relationship between the

returns and interest rate observed in our sample. The mentioned before supports the implementation of the convexity in this paper.

GARCH

Subsequently, we proceeded to the application of the GARCH model (1 1) to be able to deal with the volatility clusters throughout the time series identified in the descriptive statistics section. The table 5 displays the results.

Table 5

GARCH(1,1)					
Symbol		ω		α'	β
CFE13	***	3.74E-07	***	5.18E-01	-2.12E-02
CFE15	***	2.05E-10	***	2.88E+00	2.48E-05
CHIACB07U	***	-5.42E-07	***	2.13E-03	1.00E+00
D2_PEMEE62_170815		0.000844		1.992342	-0.773361
D2_PEMEX_061117	***	4.02E-08	***	4.34E-02	9.50E-01
D2_PEMEX4_010318	***	1.28E-08	***	1.12E-01	9.02E-01
FNCOT14	***	3.12E-11	***	3.54E-01	1.13E-01
FNCOT16	***	3.08E-07	***	5.37E-01	5.05E-01
GDFCB07	***	9.41E-06	***	6.56E-02	7.94E-01
IFHGO07	***	2.63E-05	***	2.81E+00	1.75E-02
IM170727		6.87E-10		5.99E-02	1.12E-01
IM171123		6.66E-10	**	1.04E-01	1.97E-01
IQ170629	***	8.12E-09	*	2.69E-02	-9.71E-01
IQ171109		3.32E-10		1.14E-01	8.35E-01
IS171005		-5.08E-09	***	1.629806	5.67E-01
OAXCB07U	***	1.16E-06	***	1.46E-01	8.57E-01
PEMEX12	***	4.62E-07	***	4.73E-01	4.47E-01
VERACB12-2	***	2.27E-06	***	1.94E-01	8.36E-02
VRZCB06U	***	5.50E-06	***	1.05E-01	7.73E-01
VRZCB08U	***	2.15E-06	***	2.02E-01	5.36E-01

* 90% significance level

** 95% significance level

*** 99% significance level

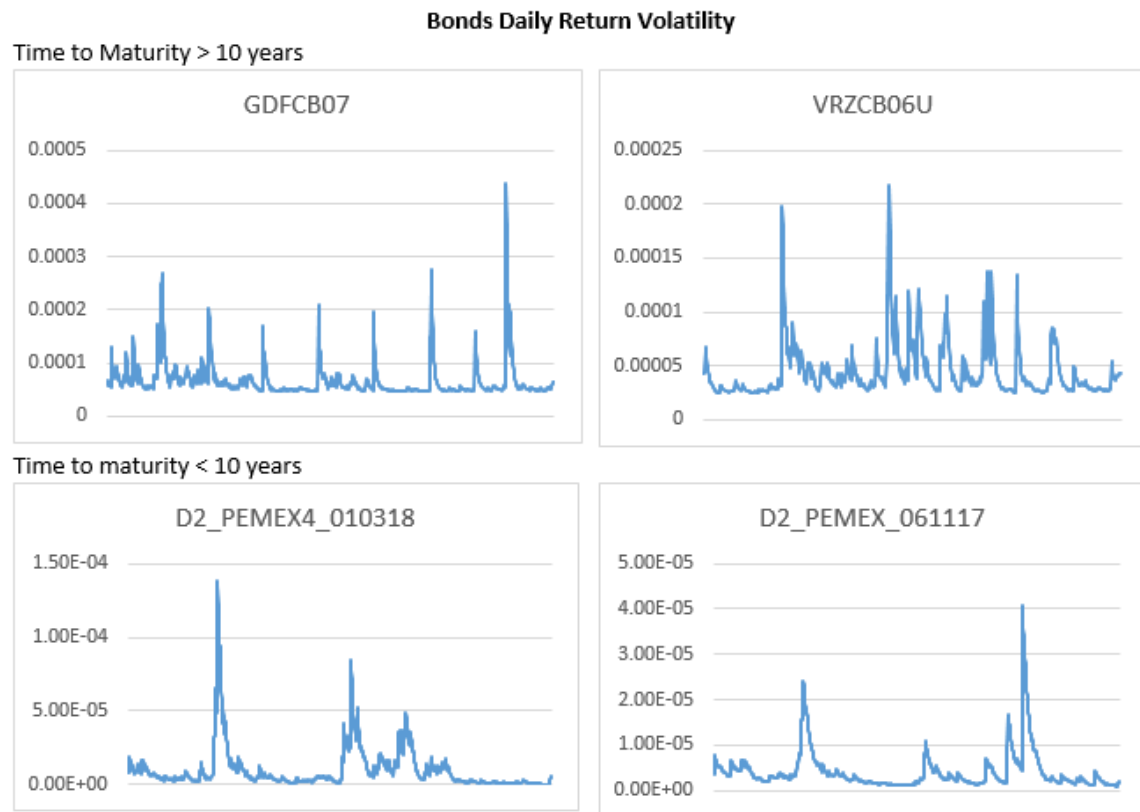
Source: own calculations

As we can appreciate most of the results seems to be consistent with the assumptions of the GARCH methodology. We only deal with inconsistency in the parameters with 5 bonds where we find negative α' 's and β 's. The sums of $\alpha + \beta$ in most of the bonds is smaller than 0 so we deal with stable GARCH (1 1) processes.

The β 's seems to be consistent with the volatility displayed in the descriptive statics section. The bonds with a higher time to maturity have a way higher volatility than the shorter bonds. So we can interpret that in this time of uncertainty in the markets the bonds with higher time to maturity

should be more sensitive to the movements in interest rates. The picture 4 has an example of two 30 years bonds and two bonds with time to maturity less than 10 years.

Picture 4



Source: own calculations

EGARCH

The EGARCH (1 1) model was run in order to measure the asymmetry factor within the volatility of the bonds. The leverage effect measure by the coefficient γ is caused by the fact that negative returns have a greater influence on future volatility than do positive returns¹¹. The results are consistent and similar to those obtained previously by the GARCH (1 1) model.

¹¹ Hotta, L. (2014) "The Leverage Effect and the Asymmetry of the Error Distribution in GARCH-Based Models: The Case Of Brazilian Market Related Series" *University of Campinas*

Table 6

EGARCH (1,1)							
Symbol	ω		α'		β		γ
CFE13	1.04E+00	***	-3.44E+00	***	6.23E+00	***	6.91E-01
CFE15	*** -5.37E+00	***	-1.30E-02	***	4.40E-02	***	7.54E-01
CHIACB07U	*** -1.15E+01		-1.24E-01	***	2.47E-01	***	-7.87E-01
D2_PEMEE62_170815	*** -1.62E+01		-5.25E-01		6.73E-01	***	-5.46E-01
D2_PEMEX_061117	*** -2.99E-02	***	-9.43E-02	***	4.22E-02	***	9.97E-01
D2_PEMEX4_010318	*** -1.25E-01	***	-7.25E-02	***	1.93E-01	***	9.88E-01
FNCOT14	-2.47E+01		-1.97E-01		1.38E-01		-7.87E-02
FNCOT16	*** -1.50E+01	***	6.15E-01	***	1.02E+00	***	-1.33E-01
GDFCB07	-1.51E-02		-3.40E-03	***	5.16E-02	***	9.99E-01
IFHGO07	*** -9.97E+00	***	-9.54E-01	***	1.60E+00	***	-7.41E-02
IM170727	*** -3.32E+01		-1.03E-01		-1.04E-01		-6.56E-01
IM171123	*** -3.41E+01	***	-3.23E-01		-7.47E-03	***	-6.32E-01
IQ170629	-2.05E+00	**	-5.46E-02	***	2.93E-01		8.94E-01
IQ171109	-6.22E+00		-5.51E-02		1.77E-01		6.74E-01
IS171005	*** -7.41E+00	***	8.00E-02	***	1.11E-01	***	5.39E-01
OAXCB07U	-7.08E-02		-6.01E-02		2.43E-01		9.89E-01
PEMEX12	*** -2.73E+00	***	1.39E-01	***	5.50E-01	***	7.85E-01
VRZCB06U	*** -9.19E-01	***	-2.68E-01	***	7.80E-02	***	9.09E-01
VRZCB08U	*** -1.06E+00	***	-1.03E-01	***	1.99E-01	***	9.06E-01

* 90% significance level
 ** 95% significance level
 *** 99% significance level

Source: own calculations

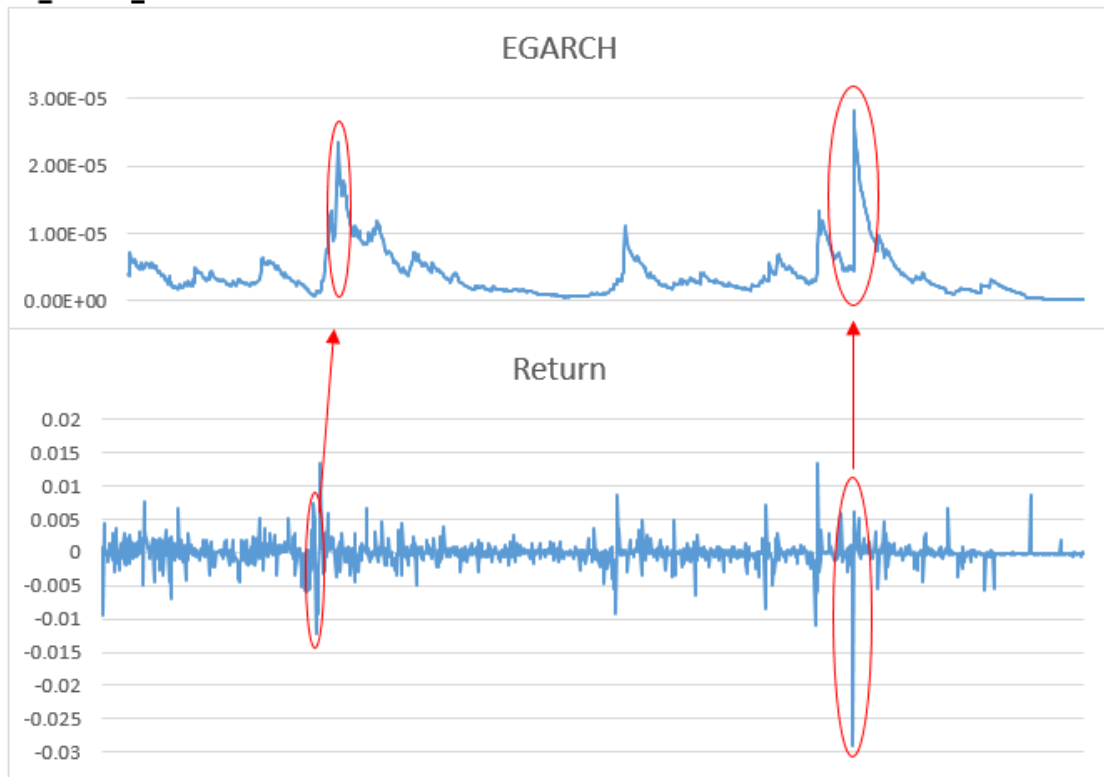
The results given by the coefficient γ proves the asymmetry in the volatility of the bonds. The coefficient γ display us that the bonds with a higher time to maturity do have a higher reaction to negative returns than the shorter bonds, presenting the asymmetric response of volatility to positive and negative returns.

Graphically the pictures 5 and 6 show us the comparison between the returns and the volatility modeled by the EGARCH (1 1) model of two bonds. One with a time to maturity less than 10 years (D2_PEMEX_061117) and a bond with a time to maturity longer than 30 years (VRZCB06U).

Picture 5

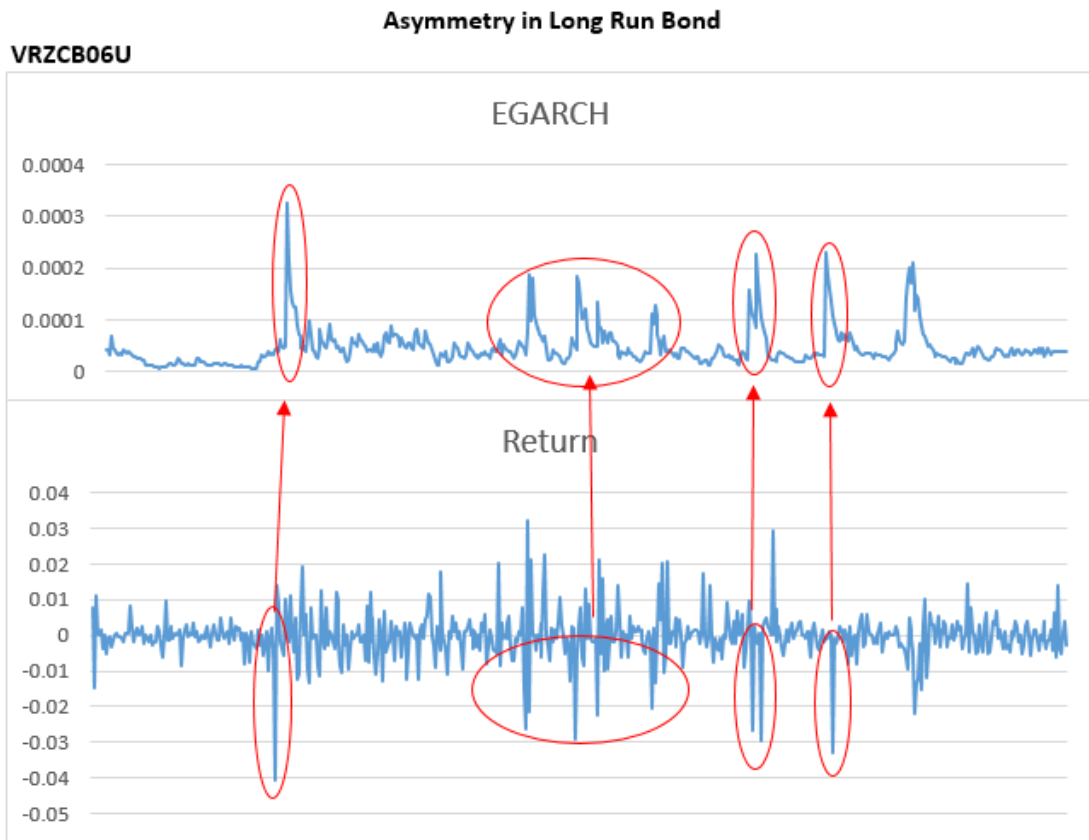
Asymmetry in Short Run Bond

D2_PEMEX_061117



Source: own calculations

Picture 6



Source: own calculations

Convexity

We continued our analysis adding the Convexity to the original model. The calculations of the two factor model is representing with the following equation:

$$Return_t = \beta + \alpha Modified\ Duration_t + C Convexity_t + \varepsilon$$

The results are presented in the table 7.

Table 7

Covexity Model						
Symbol		Coefficient α	SE α	Coecifficent c	SE c	R2(%)
CFE13	*	-0.00005	0.008900	1.66E-22	2.67E-21	0.930000
CFE15		-0.00001	0.000527	4.17E-22	1.01E-20	0.000000
CHIACB07U		0.00000	0.000021	1.5E-18	1.6E-16	0.000000
D2_PEMEE62_170815		0.00000	0.000900	1.76E-21	8.65E-20	0.010000
D2_PEMEX_061117		0.00000	0.000022	1.66E-18	7.05e-18	0.010000
D2_PEMEX4_010318	*	0.00000	0.000000	1.71E-17	1.45e-16	0.220000
FNCOT14		0.00000	0.000001	1.1E-22	1.37e-22	0.330000
FNCOT16		0.00002	0.000056	-8.64E-23	3.88e-21	0.030000
FNCOT162		0.00000	0.000002	1.05E-22	1.99e-22	0.230000
GDFCB07		-0.00001	0.000015	1.59e-17	3.05e-16	0.000000
IFHGO07		-0.00051	0.000469	3.74e-20	1.34e-19	0.190000
IM170727	*	0.00003	0.000013	1.17e-24	1.75e-22	3.610000
IM171123	**	0.00001	0.000005	-4.05e-24	3.32e-22	6.410000
IQ170629		0.00003	0.000030	1.55e-22	6.56e-22	0.860000
IQ171109		0.00002	0.000007	9.55e-23	5.38e-22	5.240000
IS171005	**	0.00003	0.000056	-2.24e-21	9.62e-19	0.080000
OAXCB07U		0.00000	0.000034	5.56e-15	4.22e-14	0.000000
PEMEX12		0.00000	0.000063	8.22e-23	7.45e-21	0.000000
VERACB12-2	**	-0.00012	0.000070	-6.68e-22	4.05e-21	0.470000
VRZCB06U		-0.00010	0.000136	1.13e-16	3.68e-15	0.090000
VRZCB08U		-0.00007	0.000050	4.99E-19	8.53E-18	0.310000

* 90% significance level

** 95% significance level

Source: own calculations

As we can see, the addition of a second factor did not improve the results, reporting several α 's equals to zero. The results were mixed, half of the bonds present a negative α , showing an inverse effect between interest rate changes and yield spread. Differing with the Duration model, the short run bonds presents this negative results.

GARCH & EGARCH Models

These inconsistencies in the results was repeated with the GARC and EGARCH models. The models were calculated with three different distributions (Gaussian, Student's T and Generalized error distribution), however due convergence issues the calculation of this models were only applied in 4 bonds.

The coefficients of this four bonds did not fulfill the GARCH process, having negative parameters. The mentioned before demonstrate us that the models did not have a proper adjustment and we cannot ensure the stationarity in both models.

VII Conclusions

This paper applies the studies of Escribano (2003 & 2004) to the Mexican bond market. The methodology applied in this paper fulfills a gap in the studies on this field in the Mexican market.

The conclusion of this paper are: (1) bonds with the shorter time to maturity tend to have an increment in the yield spread in the actual market conditions of interest rate hikes; (2) The bonds with a higher time to maturity have a way higher volatility than the shorter bonds and trend to be more sensitive to the movements in interest rates; (3) the bonds with a higher time to maturity do have a higher reaction to negative returns than the shorter bonds.

Even with these results the models proposed by Escribano are very useful since they help to quantify the effects of movements in interest rates. The inclusion of models such as GARCH and EGARCH improve the results in order that they don't have the limitation of a constant variance in the time series. The lack of convergence in the GARCH and EGARCH models including the second factor could be solved calculating an IGARCH model. Authors like Mikosc and Stărică (2004) deal with the nonstationarities in financial time series applying the model mentioned before.

The results obtained on this paper may be useful for the Mexican Federal Treasury, which is the responsible to administrate the federal resources. The measurement of the future increase in debt cost that a rise in interest rate will carry, would be an essential tool for the Treasury minister in order to reduce the indebtedness of the federal government. The recent decline in Mexico's credit rating should be an alert signal to bring debt levels to more stable positions.

The conclusions on this paper are extremely important in banking, especially when the volatility has reached alarming levels since the end of 2016. Due the actual and future market conditions it is important to measure the impact that the changes in interest could have in fixed income securities. In the last announcement of monetary policy, Banxico mentioned that for the rest of 2017 and for 2018 a moderate recovery of activity is expected global economy; However, this growth scenario continues to face downside risks, including high uncertainty due to the increase in geopolitical risks in various regions, the possible effects of recent weather events and the direction of monetary, fiscal and trade policy in the United States¹².

As a manner of outlook, it would be very useful to back test the portfolios that contain the bonds mentioned in this paper. The Society of Actuaries Pension Section, from the University of Indiana has already done this exercise suggesting the asset managers new save assets that a movement in interest rates should not have the same effects as the government securities.

¹² Banco de México (2016) "Anuncio de Política Monetaria 28-09-2017"

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