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**Metaheurística para el diseño de una red logística; Problema
de la P-mediana capacitada**

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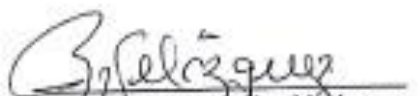
Se aprueba la Tesis:

**Metaheurística para el diseño de una red logística; problema de la P-
mediana capacitada**

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DEDICATORIA

A Meli

A ti hija, ¡sé que algún día nos volveremos a encontrar!

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Alhe, quien me ha brindado su apoyo incondicional y ha hecho suyos mis preocupaciones y problemas. Gracias por tu amor, paciencia y comprensión.

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CAPÍTULO 1. INTRODUCCIÓN

Los problemas de localización parten de la necesidad de definir la ubicación más conveniente para ubicar centros de distribución o instalaciones como: una planta de producción, CEDIS o almacenes comerciales y/o de servicios, un tiradero de basura, una estación de bomberos, una estación de policía y tránsito, un hospital e inclusive una tienda de autoservicio. Entonces, ¿cómo saber que el Centro de Distribución (CEDIS), que se pretende abrir esté en la mejor ubicación?, donde tanto el cliente como el distribuidor tengan las mejores ventajas logísticas, determinando que éste sea el más cercano al óptimo, y donde se puedan minimizar los costos en general.

El problema de la p -mediana (PMP) y p -mediana capacitada (CPMP) proporcionan una excelente opción para definir la mejor ubicación de estos centros. Básicamente el problema de la p -mediana capacitada considera una situación en la cual se requiere particionar un conjunto de clientes con una demanda en exactamente p grupos. Cada grupo estará definido no sólo por el conjunto de clientes que lo forman, sino también por la ubicación de su mediana y la capacidad de cada p .

El documento se encuentra estructurado de la siguiente manera. El capítulo 2 presenta una primera investigación titulada “A Solution Proposal for the Capacitated P-Median Problem with Tabu Search”, la cual ha sido publicada por la revista Research in Computing Science, volume 121, 2016, pages 59-67. ISSN 1870-4069. En ella se define una Metaheurística con ayuda de búsqueda TABU para el problema de la P-Mediana capacitada (CPMP), la metaheurística diseñada es evaluada con ayuda de las instancias de prueba de OR-Library y el software de optimización Lingo 16.0. Los valores obtenidos por Lingo 16.0 y los obtenidos por la metaheurística desarrollada en una plataforma Java, comparados entre si dando como resultado una ligera desviación en cuanto al tiempo computacional de la solución y al valor óptimo. En el capítulo 3 se muestra una segunda investigación titulada “Solution Search for the Capacitated P-Median Problem using Tabu Search”, la cual da continuidad a la investigación presentada en el capítulo 2. Esta segunda investigación

ofrece una mejora a la metaheurística desarrollada en la primera publicación y prueba su rendimiento a través de una instancia real en la cual era necesario rediseñar la red de distribución de una empresa del giro cárnico que actualmente cuenta únicamente con un solo centro de distribución y 17 puntos de venta. Los resultados obtenidos después del rediseño de la red de distribución ofrecen una reducción del 16.4% en lo que a costos concierne, además de lo anterior, se obtuvo una reducción muy importante en los tiempos de entrega impactando directamente al servicio al cliente y abriendo la posibilidad de expansión a nuevos mercados. Finalmente, el capítulo 4 presenta las conclusiones y recomendaciones de la investigación abordada en este trabajo de investigación.

CAPÍTULO 2. ARTÍCULO DE CONGRESO

Publicado en Research in Computing Science, volume 121, 2016, pages 59-67. ISSN 1870-4069.

Mauricio Romero Montoya, María Beatriz Bernábe Loranca, Rogelio González Velázquez, José Luis Martínez Flores

2.1 Title

A Solution Proposal for the Capacitated P-Median Problem with Tabu Search

2.2 Abstract

This work presents the Capacitated P-Median problem, which seeks to solve the optimal location of p distributors. In understanding the computational complexity of this problem, partitioning principles are applied in the process of associating a distribution center with customers to be serviced. These are treated as clusters containing each distribution center and its customers. The optimization of the implicit cost function within the partitioning (distance minimization) is performed using Tabu Search with a diversified search. A series of computational experiences is performed aided by the OR-Library test instances, where in most cases an optimal solution is reached in reasonable computing time. As Tabu Search does not obtain all expected optima, OR-Library test instances are solved with the help of Lingo 16.0 and the obtained results are compared with those resulting from Tabu Search.

2.3 Keywords

Tabu Search, capacited P-median, NP-hard.

2.4 Introduction

The location of facilities deals mainly with choosing the physical location for a set of facilities in such a way that, given a set of restrictions, the demands from a set of customers

are optimally met for some target function, from some set of candidate places. Facilities are to be understood in a broader sense, as they could be entities such as hospitals, factories, schools, ports, etc.

In this area, the P-Median and Capacitated P-Median (CPMP) problems are noteworthy due to their popularity in several areas. Here, the Capacitated P-Median problem is dealt with, as it was treated in previous work without reaching all optima.

Here, Tabu Search (TS) is introduced as an approximation tool, thus reaching the optima established in OR-Library.

This work is organized as follows: this introduction is section 1. Section 2 shows general aspects of related works. Section 3 shows the modeling leading to Section 4, which presents the approximation method and associated algorithm, section 5 shows the results of the computational experiences. Lastly, in section 7 results and conclusions are presented.

2.5 Related Works: P-Median and Capacitated Facility Location Problems

Location models are applied for specific cases, thus their structure (objectives, restrictions and variables) are case-dependent, therefore there are no generic models appropriate for all problems, and models with different objectives may provide different solutions for the same case study.

Facility location, also known as location analysis or location science, is concerned with the siting of facilities on a plane or within a network. These locations are dependent upon various attributes such as customers' demand on a facility, the cost of supplying those customers and the costs of opening facilities at potential locations.

A general description of location problems may be found in [1]. The problem dates back to Euclid and Pythagoras, due to their proposals of geometric models for distances; however literature suggests that it was emperor Constantine who pioneered its formal expression, as the location in a network was solved by means of discrete coordinates that placed the roman legions. Later, Fermat and Torricelli formulated the three node problem, which led to the Weber problem we know today (the infinite solution space Euclidian distance

minimum single site location problem). In the 19th century Sylvester posed the problem of minimizing the maximum Euclidean distance in the infinite solution space.

In the 20th century Steiner's problem is generalized and formalized. Other contributions appear in the field of logistics (Hoover and Pallander [2]), but it was Wersan, Quon and Charnes [3] who placed the problem within the framework of mathematical optimization and programming, leading to the work of Daskin [4]. Location decisions may well be the most difficult for the efficient design of a supply chain, due to their strategic nature, for it is a long-term decision, which means it should consider facility cost, which may be quite steep as happens in manufacturing facilities. Therefore, it must be concluded that it is an inflexible decision. For a deeper discussion on the relevance of location decisions the reader is advised to refer to Daskin [5].

The quality of the location decision may make the difference in the efficiency of a supply chain, affecting customer satisfaction. Let us not forget that location decisions investigate the siting of a set of facilities in order to best satisfy the needs of a set of customers (Hale and Moberg, 2003) [6] thus, a poor location decision would hinder this purpose, contradicting the definition of supply chain as proposed by Chopra and Meindl (2001) [7]. Brandeau and Chiu (1989) [8] reviewed 50 location problems that had been studied, indicating their formulations, classification and relationships, thus showing a series of optimization objectives that models may follow. Discrete models for the location in networks may be classified according to distance (Current et al. 2001) [9], distinguishing models based on maximum distance, and models based on total or average distance.

Daskin (2008) and ReVelle et al. (2008) [10, 11] reviewed and categorized the models for location problems into four classes: analytical, continuous, network and discrete. Analytical models refer to simple contrived problems. They have analytical value and are solved using classical mathematical optimization techniques but are of limited use in practice, due to their simplicity. While customer demand is most often restricted to fixed points or nodes within a region or a plane, continuous problems allow for the location of facilities anywhere within the region. These problems are known as Weber-type problems, and are often solved by means of analytical techniques. The applications and algorithmic techniques for these problems were reviewed by Drezner et al. (2001) [12, 13].

2.5.1 P-Median

The P-median problem, credited to Hakimi (1964) is an adaption of the classical Weber problem. It is particularly difficult to solve when considering problems of a more practical size where the number of variables may run in the hundreds or even thousands. Hence heuristic solution methods are often employed, as the complete enumeration time component increases exponentially with the number of binary variables to be considered (facilities and customers) [13]. The combinatorial nature of the p-median problem caused advances in solution techniques to be often made using combinations of mathematical programming and heuristic search techniques such as branch and bound and Lagrangean relaxation. This significant work, along with its results, is still often cited today by many authors. Reese (2006) [14], provides an annotated bibliography for the p-median problem which indicates that since the late 1980s there has been a significant shift from using traditional mathematical programming relaxation techniques, such as Lagrangean and surrogate relaxation, towards the use of more modern metaheuristics techniques (Bernabé 2012) [15].

2.5.2 Capacitated Facility Location Problem

The p-median and uncapacitated location problems both consider facilities with more than enough capacity or supply to provide for all their assigned customers. However, as facilities have only a finite supply or capacity, in practice this may not be the case. While most customers may be assigned to their nearest facility, when supply runs out, some customers may need to be directed –or at least partially directed – to the nearest available facility. Thus, there are two cases: first, the assignment of customers to facilities is determined via a binary constraint, to ensure a customer is served from a single facility, and second, the demand constrained may be fractional, to allow for the demand to be satisfied from one or more facilities. As a direct consequence, the capacitated p-median and the capacitated facility problems have been the interest of studies. The capacity constraints result in greater theoretical difficulty for the solution of these problems, and are generally

NP-hard. The contributions to solving the capacitated p-median problem are primarily metaheuristic or hybrid based.

The capacitated facility location problem may be regarded as either deriving from the the p-median problem by including fixed opening costs for facilities or, even better, as extending the uncapacitated facility location problem by considering each facility to have a limited supply or capacity constraint. It is referred to be NP-hard, and Kariv and Hakimi (1979) [16, 17] and Garey and Johnson (1979) [18] are often cited, even though neither considered this problem. However, the problem is modeled on a network, and is therefore likely to be at least as hard as the p-median problem. Heuristic algorithms are often presented as solution techniques. ReVelle and Eislet (2005) [19] pointed out that the introduction of capacity constraints effectively eliminates the property that all demand from a given customer is to be assigned to a single facility, and thus makes the problem much more difficult to solve.

2.6 Metaheuristics for Capacitated P-Median CPMP

Many attempts have been made to solve both capacitated and uncapacitated P-median. Two articles are of interest regarding Tabu Search, as this approximation to the capacitated p-median problem is solved by means of TS.

A Tabu Search procedure for the CPMP was developed by Sörensen (2008) [20], which managed to produce optimal solutions for all the small to medium sized OR-Library test problems, save two. Even though their paper does not identify said instances, relative errors were reported to be less than 0.1%, which represents an improvement over any previous work for CPMP. This work both backs up the conclusions by Arostegui et al. (2006) [21] and suggests that Tabu Search may well be a dominant methodology to use when solving the CPMP.

2.6.1 Capacitated P-Median Model

The CPMP, also known as the Capacitated Clustering Problem, the Capacitated Warehouse Location Problem, the Sum-of-Stars Clustering Problem and some others, is a classical location problem that finds applications in many practical situations. It may be described as: given a set of n points (customers), each of them with a known demand, it is required to find p medians (centers) and assign each point to exactly one median such that the total distance of assigned points to their corresponding medians is minimized, and the capacity limit on the medians may not be exceeded. Several heuristics and metaheuristics have been proposed for these problems, known to be NP-hard [18]. Osman and Christofides [22] propose a simulated annealing and tabu search method. Maniezzo [7] present a bionomic algorithm for the solution of this problem. Lorena and Senne [6] explore local search heuristics based on location-allocation procedures and Lorena and Senne [25] use column generation to CPMP.

The CPMP considered in this paper is modeled as the following binary integer programming problem:

$$z = \min \sum_{i \in N} \sum_{j \in N} d_{ij} x_{ij} \quad (1)$$

$$\text{subject to } \sum_{j \in N} x_{ij} = 1, \forall i \in N \quad (2)$$

$$\sum_{j \in N} x_{jj} = p \quad (3)$$

$$\sum_{i \in N} q_i x_{ij} \leq Q x_{jj}, \forall j \in N \quad (4)$$

$$x_{ij} \in \{0,1\}, \forall i \in N, \forall j \in N \quad (5)$$

Where: $N = \{1, \dots, n\}$ is the index set of points to allocate and also of possible medians, where p medians will be located; q_i is the demand of each point and Q the capacity of each possible median; d_{ij} is a distance matrix; x_{ij} is the allocation matrix, with $x_{ij} = 1$ if point i is allocated to median j , and $x_{ij} = 0$, otherwise; $x_{jj} = 1$ if median j is selected and $x_{jj} = 0$, otherwise.

Target function (1) minimizes the sum of distances between all demand nodes. Constraint (2) ensures each node is assigned to exactly one facility. Constraint (3) sets the number of medians to be located. Constraint (4) imposes that a total median capacity must be respected, and (5) provides the integer conditions.

Clustering Search Partitioning (CSP) was used to assign clients to suppliers. This uses clustering to detect promising areas in the search space. It is of particular interest to find out these areas as soon as possible, in order to change the search strategy over them. In CSP, a clustering process is executed simultaneously with the Tabu Search algorithm, identifying groups of individuals that call for special interest.

The CS tries to locate promising search areas by framing them by clusters. Clusters may be defined as $G=\{c; r; s\}$ where c is the center, r the radius of the area, and s is a search strategy associated with the cluster.

The center c is a solution that represents the cluster, identifying the location of the cluster inside of the search space. The radius r establishes the maximum or minimum distance, starting from the center, that a solution can be associated to the cluster. The search strategy s is a systematic search intensification, in which solutions of a cluster interact among themselves along the clustering process, generating new solutions.

2.7 Tabu Search for Capacitated P-Median

This approximation relies on traditional Tabu Search, which generates a random initial solution. The parameters here have been defined as: 1) `numberOfIterations` (global iterations to be performed by the algorithm) 2) `perturbationIterations`, (determines the number of times a solution may be accepted according to the solution of the previous iteration. When the counter `perturbationCounter` crosses this limit, a new random solution is generated, and search is restarted).

The `module` parameter determines the frequency with which in Vicinity 1 (Algorithm 2) is to be used instead of Vicinity 2 (Algorithm 3), as can be seen in the first conditional within the main while loop. When the cost given by the vicinity functions improves on the best

known cost (bestCost), this will be replaced, along with the solution configuration (bestSolution). The last step updates the tabu lists and global iteration counter.

Algorithm 1. Tabu Search (PSEUDOCODE):

```

READ numberOfIterations
READ perturbationIterations
READ module
SET iterationCounter and perturbationCounter to 0
SET initialSolution and bestSolution to currentSolution

WHILE iterationCounter < numberOfIterations
    SET currentCost to cost of currentSolution
    IF iterationCounter % module = 0
        CALL Neighbor1 with currentSolution RETURNING newCost
    ELSE
        CALL Neighbor2 with currentSolution RETURNING
newCost
    END IF
    IF newCost > currentCost
        INCREMENT perturbationCounter
    END IF
    IF newCost < bestCost
        SET bestSolution to newSolution
        SET bestCost to newCost
    END IF
    SET currentSolution to newSolution
    SET currentCost to newCost
    IF perturbationCounter > perturbationIterations
        CALL RestartSolution RETURNING newSolution
        SET currentSolution to newSolution
        SET perturbationCounter to 0
    END IF
    CALL UpdateTabuLists RETURNING updatedTabuLists
    INCREMENT iterationCounter
END WHILE
RETURN bestSolution

```

CPMP is a problem in combinatorial optimization, classified as NP-Hard, and the tabu search metaheuristic has proven efficient in searching for solutions for a wide variety of problems of this kind, such as the quadratic assignment problem, the traveling salesman problem, or p-median search, among others. The tabu search metaheuristic uses different

intelligent search schemes, like the short and long term memory use, keeps a list of tabu solutions that should not be visited because they lead to trajectories that have proven not to be good solutions; these are tools in the search for artificial intelligence.

2.8 Computational Experience

Table 1 gathers the attained results for two kinds of tests with the Tabu Search approach: 1) varying parameters (random) and 2) using default set parameters and, on the other hand, the results obtained by the Branch & Bound method in Lingo 16 are presented.

It can be seen that varying parameters gives better results, and the OR-Library [26] demanded optima are approached. On the other hand, the results obtained are better than those reached in previous work [27].

Table 1 Results for Capacited P-Median with TS

I	Capacited Q=120 Random						Default			Lingo 16		
	BV	C	P	RC	T	GAP	DC	T	GAP	OPT	T	OPT
1	713	50	5	717	48340	0.56	722	10040	1.26	713		7.77
2	740	50	5	740	50933	0	740	9950	0	740		1.05
3	751	50	5	752	72445	0.13	772	9810	2.8	751		2.34
4	651	50	5	653	53402	0.31	654	10030	0.46	651		1.18
5	664	50	5	664	50073	0	664	9950	0	664		2.80
6	778	50	5	778	24426	0	778	9930	0	778		1.55
7	787	50	5	796	52371	1.14	810	9820	2.92	787		63.02
8	820	50	5	826	54607	0.73	851	9690	3.78	820		236.96
9	715	50	5	718	23938	0.42	721	9840	0.84	715		9.95
10	829	50	5	844	47455	1.81	844	9810	1.81	829		129.25
11	1006	100	10	1054	3E+05	4.77	1074	11900	6.76	1006		119.65
12	966	100	10	978	2E+05	1.24	983	11900	1.76	966		219.48
13	1026	100	10	1032	1E+05	0.58	1055	12050	2.83	1026		27.22
14	982	100	10	1012	3E+05	3.05	1027	12110	4.58	982		1190.93
15	1091	100	10	1133	7E+05	3.85	1160	12010	6.32	1091		711.64
16	954	100	10	975	2E+05	2.2	985	12120	3.25	954		108.53
17	1034	100	10	1077	64331	4.16	1072	11960	3.68	1034		112.21
18	1043	100	10	1086	67329	4.12	1088	12040	4.31	1043		543.21
19	1031	100	10	1057	63633	2.52	1049	11780	1.75	1031		380.67
20	1005	100	10	1133	66159	12.7	1146	11850	14.03	1005		48439.28

In the table 1, the notation is the next: I=Instance, BV is the best known value published in the OR-Library, C are the n Customers, P are the medians, RC are the values in the experiment when we were varying parameters (random), T (sec), is the execution time for each test, The GAP is the difference between the best cost and the obtained value by the approximation method, DC are the default parameters by default with TS, OPT are the values obtained with optimization software Lingo 16, T OPT is the run time for each instance in Lingo 16. Tests were performed on a PC with 6GB RAM and a second generation i5 processor.

2.9 Conclusions

Throughout this work the extensive literature, efforts and applications surrounding the Capacitated P-Median problem have been confirmed.

Our objective is to obtain all optima dictated by the OR-Library, and at this point in the computational experience, it is assumed that satisfaction of optima is possible by adjusting the parameters in Tabu Search in such a way that the design of factorial statistical experiments becomes necessary.

In this work the robustness of the tabu search performed on the 20 OR-Library instances has been proven and compared with the results from Lingo, with an acceptable GAP, as shown in table 1.

The contribution from this work is the design of a sequential program based in object oriented programming and a data structure that allows for covering trajectories that partially explore the set of feasible solutions, known to have a combinatory explosion as the size of the problem increases even in moderate cases. This Tabu Search proposal is competitive even with the implementation of Lingo 16, a professional version without restrictions that is recommended for large scale testing.

Lastly, the capacitated P-median is part of the localization-assignment problems which define the strategy of an organization, that is, they are long-term decisions, and a good decision in the distribution and location of facilities has positive implications in the efficiency of the supply chain and the cost competitiveness of products and services

reaching the customer, thus in general being benefic for the population sector that demands said product or service. In this point, as a future development from this work, a series of applications with both TS and Lingo 16 is to be developed, in particular for the meat industry.

CAPÍTULO 3. ARTÍCULO DE REVISTA

Aceptado para publicarse en International Journal of Combinatorial Optimization Problems and Informatics.

3.1 Title

Solution Search for the Capacitated P-Median Problem using Tabu Search

3.2 Abstract

Capacitated p-median problem (CPMP) is an important facility location problem, where Capacitated p-medians are economically selected in order to serve a set of demand vertexes in such a way that the total demand placed on each of the candidate medians does not exceed their capacity. This work presents an efficient Tabu-search based metaheuristic to solve the CPMP. Said metaheuristic is tested by means of computational experiments aided with the OR Library test instances, and in most cases the optimum value is reached in reasonable computing times. The obtained results are compared and evaluated with the aid of Lingo 16.0. Lastly, an actual case is solved in which the objective is presenting new locations for a meat-product company's distribution centers.

3.3 Keywords

Tabu Search, Metaheuristic, Capacited p-median, Np-Hard

3.4 Introduction

Location problems arise from the need to define the most convenient location to place facilities such as distribution centers (DC), production plants, commercial warehouses, service centers, trash dumps, fire or police stations, hospitals or supermarkets.[20] Yet, how can one ensure that the DC is placed at the best location? By the best location, we

mean that both customers and suppliers have the best logistic advantages, that is closest to the optimum and that costs in general are minimized.

The p-median problem (PMP) and the Capacitated p-median problem (CPMP) are excellent options to define the best location for these centers. The Capacitated p-median basically considers a situation in which it is required to partition a set of clients with demand into exactly p groups. Each group is defined not only by the set of clients in it, but also by the location of their median and each p's capacity. A clear representation of this problem is presented in fig. 1.

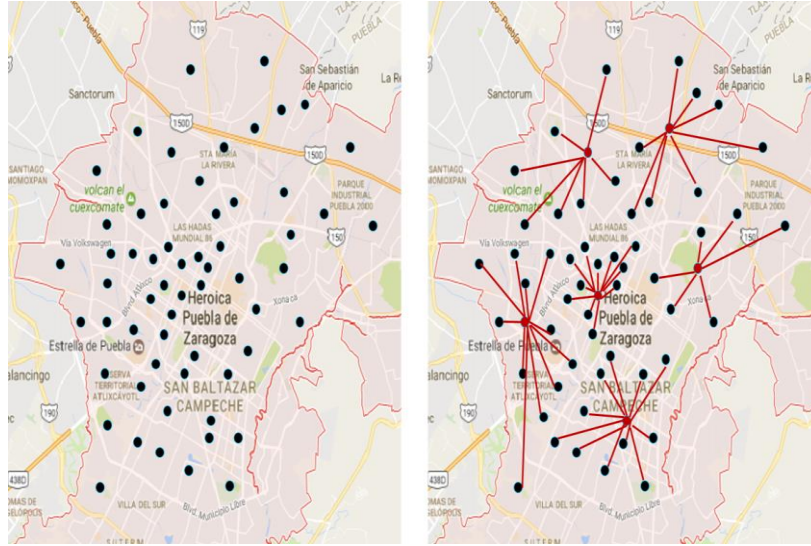


Figure 1 Capacitated P-Median Problem (CPMP)

The present work is organized as follows: Section 1 presents the introduction. Section 2 includes a brief description of the Capacitated P-Median Problem and its mathematical formulation, In section 3 the approximation method and the algorithm designed under Tabu search are presented, Section 4 shows the result of the computational experiments, section 5 shows the algorithm applied to an actual life case. Lastly, in section 6 the conclusions from this work are presented.

3.5 Problem description

The CPMP problem is an adaptation of the PMP which was developed by Hakimi (1946). It is not easy to solve the CPMP problems by any of the linear programming traditional methods when some applications are considered and at the same time are compound of a large number of variables and, therefore, they have large solution spaces. Due to this, it is necessary to consider other possible solution methods such as heuristic ones. The combinatorial nature of the p-median problem and the advancements which were added to the solution techniques, both allow the promotion of excellent outcomes which are very close to the optimum result. These outcomes are very often calculated from the combination of mathematical programming and some others searching and heuristic techniques such as, the tabu search and the lagrangian relaxation [22].

Informally, the objective of the p-median problem is the determination of P locations for facilities from a set of n (nsp) previously defined candidates in such a way that the demand set is met and the sum of the distances between the demand points and the selected locations is minimized.

Formally, it is assumed that each vertex in a graph is a potential median. The p-median problem maybe formally defined as follows:

Let $G=(V,A)$ be a simple directed graph where V represents the vertexes and A represents the arcs. The objective is to find the vertex set V_p (set of medians) with cardinality P, such that: a) the sum of the distances between the remaining vertexes and the selected vertex V_p in $\{V-V_p\}$ (the demand set) is minimized, and b) all demand points are met without violating the capacity restrictions for the located median facilities.

When compared with the p-median problem, the Capacitated P-median has the added restrictions that: (1) each facility placed can only satisfy a limited demand (capacity restriction) and (2) each demand point should have its demand met by its respective median- selected facility.

The capacitated p-median problem is presented next, in terms of integer programming [15]. The capacitated p-median problem (CPMP) aims to solve the optimal location for p facilities, considering both distances and service capacities offered by each facility. Total

service demanded by the vertexes identified in each p-median cluster may not exceed the facility's capacity. The mathematical formulation follows:

$$y_{ij} = \begin{cases} 1 & \text{if the median is placed at location } j \in J \\ 0 & \text{is not assigned in other cases} \end{cases}$$

$$X_{ij} = \begin{cases} 1 & \text{if client } i \text{ is assigned to location } j \\ 0 & \text{is not assigned in other cases} \end{cases}$$

$$\text{Min } \sum_{i \in N} \sum_{j \in J} w_i c_{ij} X_{ij} \quad (1)$$

Subject to:

$$\sum_{j \in J} X_{ij} = 1 \quad \forall i \in N \quad (2)$$

$$\sum_{j \in J} Y_j = P \quad (3)$$

$$\sum_{i \in J} w_i X_{ij} \leq q_j Y_j \quad \forall i \in N, j \in J \quad (4)$$

$$X_{ij} \in \{0,1\}, Y_j \in \{0,1\} \forall i \in N, j \in J \quad (5)$$

Target function (1) minimizes the sum of distances between all demand nodes. Constraint (2) ensures each node is assigned to exactly one facility. (3) restricts demand nodes to be assigned. (4) ensures that one client is assigned to only one selected median. (5) are binary restrictions.

As can be seen, the difference between the classical P-median problem and the CPMP lies in the fact that CPMP sets a fixed amount of service or product that facility j provides, which is very important from the economic point of view. On one hand, CPMP allows the decision maker to locate facilities regardless of their capacity, while covering all or part of the demand that corresponds to the capacity of each facility.

Much research has been done to solve the p-median and CPMP problems, yet one of the biggest challenges is dealing with the great amounts of data needed to provide good quality solutions within reasonable computing time. A proposal to solve the CPMP for large instances is the inclusion of Tabu search embedded with dual relaxation, with the added

characteristic that a limit on the quality of the solutions while the dual relaxation of the problem is solved heuristically. [14]

3.5.1 Metaheuristics for Capacitated P-Median CPMP

The combinatorial nature of the p-median problem caused advances in solution techniques to be often made using combinations of mathematical programming and heuristic search techniques such as branch and bound and Lagrangean relaxation [18]. Several efforts have been made in order to solve the p-median and capacitated P-median problems. However, two articles are of the greatest interest when Tabu search is involved, because the methods developed to solve the p-median problem are solved by means of Tabu search (TS). A TS procedure for CPMP was developed by Sörensen (2008) [21] that achieved optimal solutions for all the small and medium sized instances in the OR- Library, except for two. Even though the paper fails to identify said instances, it is established that the relative errors were under 0.1%, which represents an improvement over all previous work for the CPMP. This work backs the conclusions by Arostegui et al (2006) [3] and establishes that TS may well be a dominant methodology to use when solutions for the CPMP are sought. There are algorithms of solution that go from the search of the optimum making simplifications of the model as it can be seen in COBRA [5], to the application of metaheuristics, some examples of these are genetic algorithms, simulated annealing, clustering tools, among others. It is important to bring out that all these algorithms search the optimum. Taking into account the clustering method, the methods of the p-median have been created within the data mining area to detect patterns in large scale of data as it can be seen in regions which are densely populated in an Euclidean space. One of the challenges is to use those large amounts of data properly to obtain quality in the solutions in a reasonable computing time [9].

In this context, it is possible to see that most of those models are formulated with Mixed Integer Linear Programming (MILP); this is done because the exact ways to solve this type of problems in medium sizes need powerful computers; so the heuristic approaches are used to solve the real problems. The main techniques used are patterns of decomposition,

hierarchical, Lagrangian relaxation, branching and dimensioning models and some metaheuristics such as simulated annealing and genetic algorithms [19]. Hindi and Basta in 1994 used the branching and dimensioning techniques to work the same Geoffrion & Graves model; they also added the limited capacity constraint and calculated the lower limits through structured transformations (Hindi and Basta, 1994). In the same area, there were great results when p-median and simulated annealing were mixed. The effectiveness of this type of methods can be seen clearly through the tests done with the instances of OR-Library [2]. Lately, it can be found an excellent review of this literature on p-median [14]. About partitioning, considered as a p-median problem, the correct grouping is given when the elements of the partition meet the geometric criterion of compactness, which it is achieved when the distance is minimized between the distribution centers and the clients (centroids and geographic objects) [4]. The Tabu Search (TS) meta-heuristic has proved highly successful for solving combinatorial and nonlinear problems. A key aspect of TS consists of using adaptive forms of memory to forbid the search process to revisit solutions already examined unless the trajectory to reach it is different.[10]. Herda, M., & Haviar, M. suggests two ways of combining a genetic algorithm (GA) with integer programming to improve the quality solution of the CPMP [12]. For other side Comber & Sasaki describes the development and application of a modified grouping genetic algorithm (GGA) used to identify sets of optimal ambulance locations [6]. Osman and Christofides [17] propose a simulated annealing and tabu search method. Finally, good results are obtained when using 3 different types of search for the CPMP, In the first one solutions are obtained with Scatter Search whereas the second algorithm uses Path Relinking. A third approach that combines Path Relinking with Scatter Search is also analyzed. The GRASP methodology is used to generate the initial Reference Set both for Scatter Search and Path Relinking [7].

3.6 Tabu Search for Capacitated P-Median

Tabu Search emerged from diverse works published in the late 70's. Despite the fact that the main concepts and strategies behind it already existed, it was in 1989 when the name

and methodology were formally established by Fred Glover and Ma-nuel Laguna in the homonymous book Tabu Search [8].

This approximation relies on traditional Tabu Search, which generates a random initial solution. The parameters here have been defined as: 1) numberOfIterations (global iterations to be performed by the algorithm) 2) perturbationIterations, (determines the number of times a solution may be accepted according to the solution of the previous iteration. When the counter perturbationCounter crosses this limit, a new random solution is generated, and search is restarted).

The module parameter determines the frequency with which in Vicinity 1 (Algorithm 2) is to be used instead of Vicinity 2 (Algorithm 3), as can be seen in the first conditional within the main while loop. When the cost given by the vicinity functions improves on the best known cost (bestCost), this will be replaced, along with the solution configuration (bestSolution). The last step updates the tabu lists and global iteration counter.

Algorithm 1. Tabu Search (PSEUDOCODE):

```
READ numberOfIterations
READ perturbationIterations
READ module
SET iterationCounter and perturbationCounter to 0
SET initialSolution and bestSolution to currentSolution

WHILE iterationCounter < numberOfIterations
    SET currentCost to cost of currentSolution
    IF iterationCounter % module = 0
        CALL Neighbor1 with currentSolution RETURNING newCost
    ELSE
        CALL Neighbor2 with currentSolution RETURNING
newCost
    END IF
    IF newCost > currentCost
        INCREMENT perturbationCounter
    END IF
    IF newCost < bestCost
        SET bestSolution to newSolution
        SET bestCost to newCost
    END IF
    SET currentSolution to newSolution
    SET currentCost to newCost
```

```

    IF perturbationCounter > perturbationIterations
        CALL RestartSolution RETURNING newSolution
        SET currentSolution to newSolution
        SET perturbationCounter to 0
    END IF
    CALL UpdateTabuLists RETURNING updatedTabuLists
    INCREMENT iterationCounter
END WHILE
RETURN bestSolution

```

The CPMP is a combinatorial optimization problem belonging to the NP-Hard category[13,15]. The Tabu search metaheuristic has proven effective in searching for solutions for a great variety of problems of this kind, such as the quadratic assignment problem, the traveling salesman problem or the p-median problem, amongst others. The Tabu search metaheuristic uses different intelligent search schemes such as the use of short and long term memories, keeping a list of Tabu solutions to be avoided by virtue of their leading to trajectories where it has been proven there are no good solutions; these are search tools of artificial intelligence.

3.7 Results

The algorithm here presented was validated using the instances drawn from the O-Lib [16] for CPMP problems, where the range of instances runs from $n=100$ to $n=900$, and $p=5$ to $p=20$. The files of the instances, which were downloaded directly from the O-Lib files, are encoded in order to decode the information. It was also necessary to apply the Floyd algorithm. As a result of this activity, the necessary dissimilarity matrices were obtained to solve the CPMP instances. Code was developed in Java and tested with a PC with 6GB RAM and a second generation i5 processor. Each instance from the O-Lib was executed a total of 50 times in order to obtain registers for the best solution and computing times. The O-Lib instances were also solved with the Lingo 16.0 optimization software and results were later compared, which showed that the solution times for each of the instances was short. However, the solutions obtained by the algorithm were not always near the optimum.

On table 1, the notation is the next: I=Instance, C=(n) are the Customers, R COST are the values in the experiment when parameters varied (random), T (seg), is the execution time for each test, OPT are the values obtained with Lingo 16.0, T OPT is the run time for each instance in Lingo 16.0, D COST are the default parameters with TS, and GAP is the difference between the best cost and the obtained value by the approximation method.

Table 2 Results for Capacited P-Median with TS

I	Capacited Q=120 Random					Default				Lingo 16	
	BV	C	P	RC	T	GAP	DC	T	GAP	OPT	T OPT
1	713	50	5	717	48340	0.56	722	10040	1.26	713	7.77
2	740	50	5	740	50933	0	740	9950	0	740	1.05
3	751	50	5	752	72445	0.13	772	9810	2.8	751	2.34
4	651	50	5	653	53402	0.31	654	10030	0.46	651	1.18
5	664	50	5	664	50073	0	664	9950	0	664	2.80
6	778	50	5	778	24426	0	778	9930	0	778	1.55
7	787	50	5	796	52371	1.14	810	9820	2.92	787	63.02
8	820	50	5	826	54607	0.73	851	9690	3.78	820	236.96
9	715	50	5	718	23938	0.42	721	9840	0.84	715	9.95
10	829	50	5	844	47455	1.81	844	9810	1.81	829	129.25
11	1006	100	10	1054	3E+05	4.77	1074	11900	6.76	1006	119.65
12	966	100	10	978	2E+05	1.24	983	11900	1.76	966	219.48
13	1026	100	10	1032	1E+05	0.58	1055	12050	2.83	1026	27.22
14	982	100	10	1012	3E+05	3.05	1027	12110	4.58	982	1190.93
15	1091	100	10	1133	7E+05	3.85	1160	12010	6.32	1091	711.64
16	954	100	10	975	2E+05	2.2	985	12120	3.25	954	108.53
17	1034	100	10	1077	64331	4.16	1072	11960	3.68	1034	112.21
18	1043	100	10	1086	67329	4.12	1088	12040	4.31	1043	543.21
19	1031	100	10	1057	63633	2.52	1049	11780	1.75	1031	380.67
20	1005	100	10	1133	66159	12.7	1146	11850	14.03	1005	48439.28

3.8 Test with real instance

The studied company has two main business areas, production and commercialization of meat products. All raw materials arrive to the distribution center located in the mexican city of Puebla, in the state of the same name. From this location, they are re distributed and / or are processed to be later distributed to each of 17 different sale points, other than this distribution center, which implies major costs in transportation as well as some other problems. Table 2 shows the capacity and demand for each business unit. A fixed operation

cost of \$30000.00 is considered for all company's facilities. Figure 2 shows the distribution network originally operated by the company with only one distribution center.

Table 3 Current status of the company's business units.

Location	Capacity (tons)	Demand (tons)
Puebla, Puebla.	60	7
San Martin Texmelucan, Puebla.	30	7
Ciudad de México.	30	5
Guadalajara, Jalisco.	30	4
Aguascalientes, Aguascalientes.	30	4
Tampico, Tamaulipas.	15	2
Actopan, Hidalgo.	18	4
Tierra Blanca, Veracruz.	30	2
Acayucan, Veracruz.	30	2
Tulancingo, Hidalgo.	17	3
Tonalá, Chiapas.	30	4
Villaflora, Chiapas.	30	3
Pijijiapan, Chiapas.	30	2
Arriaga, Chiapas.	30	6
Mapastepec, Chiapas.	30	2
Monterrey, Nuevo León.	30	4
Reynosa, Tamaulipas.	30	5
Torreón, Coahuila.	30	4

The current monthly total operation cost for the distribution system of the company is \$160,066.00MXN, including the fixed monthly operation cost for the distribution center in Puebla, of \$30,000.00MXN.

The distance matrix (in km) between all 18 cities that could potentially be distribution centers is considered. Distances were found using Google Maps, considering round trips. The corresponding files with all data (capacity, demand, fixed costs and distance tables). To generate a proposal for a new distribution strategy for the company, two P values are considered: P=2 and P=3. Therefore, two instances for the Capacitated P-Median problem are used to run both the proposed algorithm and Lingo 16.0.

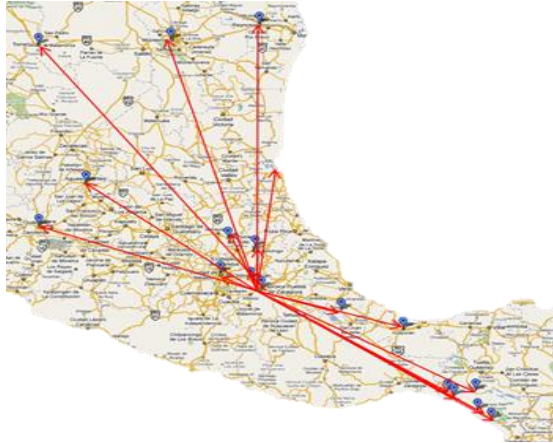


Figure 2 Current distribution network

First instance, 2 Distribution Center proposal

In this first instance, two distribution centers, $P=2$, are proposed, instead of the current one center system. The obtained results, and the computing run times for both solution methods are presented in Table 3. The generated distribution strategy thanks to the results derived from the Tabu search algorithm is shown in Figure 3.

Table 4 Results for real data with two DC

P=2	Sol.	T (sec.)
Tabu Search	138,198	7.43
Lingo 16.0	138,198.3	0.39



Figure 3 Proposed distribution for $p=2$

The study of the obtained data from the Tabu search algorithm is of great interest on its own, and for this particular company, for when total transportation costs are analyzed, for one distribution center results in costs of \$160,066.00, versus total costs of \$138,198.3 for two distribution centers, representing a reduction of 13.7% in transportation costs, In this proposal, both Puebla and the city of Arriaga are enabled as distribution centers, the latter servicing mostly the south-east of the country, while Puebla serves the center and north of the country. This initial solution is good for it implies considerable cost reductions, however it is plain to see that distances between the distribution center and locations such as Monterrey are still very large, so a second instance is tried, contemplating the possibility of two distribution centers in addition to the one already in operation.

Second instance, 3 Distribution Center proposal

In a second instance, the impact of opening three operation centers ($p=3$) in the distribution related costs is analyzed for the company's products. The obtained results and computing run times for both models are shown in Table 4, while the 3 distribution centers assigned by the Tabu search algorithm to serve the different sales points are shown in Figure 4.

Table 5 Real data results with 3 DC

P=3	Sol.	T (sec)
Tabu Search	133,824	7,345
Lingo 16.0	133,824.7	0.23



Figure 4 Proposed distribution network for $p=3$

Analysis of the resulting data shows total transportation costs of \$133,824.7, that is, a reduction of 16.4% when compared to the current monthly costs, and 3% when compared to the first instance. This seemingly small cost reduction comes with other substantial benefits, such as an improvement in customer service, for the previous delivery time is 4 to 6 days, while the new estimate is 3 to 4 days, which will benefit the perception of the company by the customer. It is also estimated that the product availability will increase, as the demand projections will be made in function of the sector to be supplied with these products, so demand will be disaggregated amongst them, while each distribution center will be able to place orders from suppliers, eliminating duplicated buying orders and the delay in their processing.

3.8 Conclusions

The research described in the present work has been motivated by the need to more efficiently find solutions to the CPMP, as shown by the robustness of the designed algorithm, validated by means of the 20 OR-Library instances, which have been compared with the results obtained with Lingo, with an acceptable GAP, as shown in Table 1. It is well known that CPMP is part of the location-allocation problems, which define the strategy for an organization, that is, they are long term decisions, and a good decision in the distribution and placement of facilities has positive implications in the efficiency of the supply chains, as well as in the cost competitiveness of the products or services reaching customers, thus generally benefiting the population sector that demands the product or service.

Given the results for the real instance, the implementation of these strategies in the industrial sector becomes fundamental in ensuring competitiveness, and with it, the survival of the organizations.

Lastly, it has been shown that PMP and CPMP are exactly the same when for CPMP an unlimited capacity is considered in the restrictions.

CAPÍTULO 4. CONCLUSIONES Y RECOMENDACIONES

La investigación descrita en este trabajo ha sido motivada por la necesidad de encontrar soluciones de una manera más eficiente al CPMP, y prueba de ello es la robustez del algoritmo diseñado y validado con ayuda de las 20 instancias de OR-Library, las cuales se han comparado con los resultados obtenidos con Lingo y se consiguió un GAP aceptable como se muestra en la tabla 1. Es bien conocido que el CPMP forma parte de los problemas de localización-asignación, los cuales definen la estrategia de una organización, es decir que son decisiones a largo plazo y una buena decisión en la distribución y ubicación de las instalaciones tiene una implicación positiva en la eficiencia de la cadena de suministro, así como en la competitividad de los costos de los productos o servicios que llegan al cliente beneficiando en general al sector de la población que demanda el producto o servicio.

Dados los resultados obtenidos en la instancia real, la implementación de este tipo de herramientas dentro del sector industrial se vuelve fundamental para asegurar la competitividad y supervivencia de las organizaciones.

Por último, derivado de las experiencias computacionales desarrolladas se ha planteado un trabajo futuro, donde en el cual se desea comprobar la hipótesis que el PMP y el CPMP son exactamente iguales cuando para el CPMP se considera dentro de sus restricciones una capacidad ilimitada.

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ANEXOS

Anexo A. Certificado de participación en el congreso internacional MICA I 2016

Certificado otorgado por The Mexican Society for Artificial Intelligence (SMIA) y el Instituto Tecnológico de Cancún en el marco del Congreso “MICA I 2016” por la participación con la ponencia “A solution proposal for the Capacited P-Median Problem with Tabu Search”



Figure 5 Certificado de participación en el congreso internacional MICA I 2016

Anexo B. Correo de confirmación de publicación de artículo de revista arbitrada

El artículo actualmente se encuentra publicado por la revista Research in Computing Science, ISSN 1870-4069 (formerly ISSN 1665-9899), Indexing: DBLP, LatIndex, la URL en donde puede visualizarse y descargarse el artículo completo es la siguiente: http://www.rcs.cic.ipn.mx/rcs/2016_121/A%20Solution%20Proposal%20for%20the%20Capacitated%20P-Median%20Problem%20with%20Tabu%20Search.pdf

La Figura 6 muestra el correo de confirmación de la publicación del artículo.

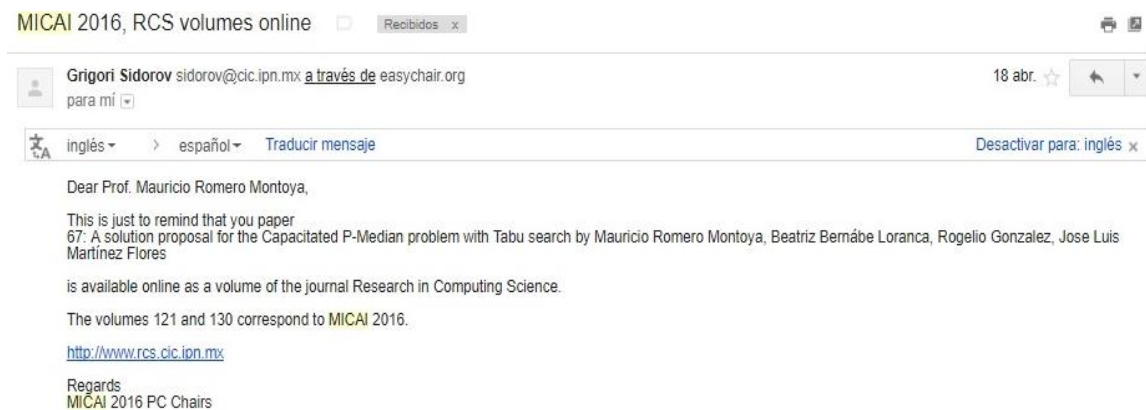


Figure 6 Correo de confirmación de publicación de artículo de revista arbitrada

Anexo C. Carta de aceptación de artículo en revista de índice CONACYT

El artículo de revista fue sometido a evaluación el día 13 de Mayo de 2017 al International Journal of Combinatorial Optimization Problems and Informatics, revista que se encuentra indexada CONACYT, este índice asegura la más alta calidad para los productos de investigación que son publicados dentro de esta revista. La Figura 7 muestra la carta de aceptación del artículo.

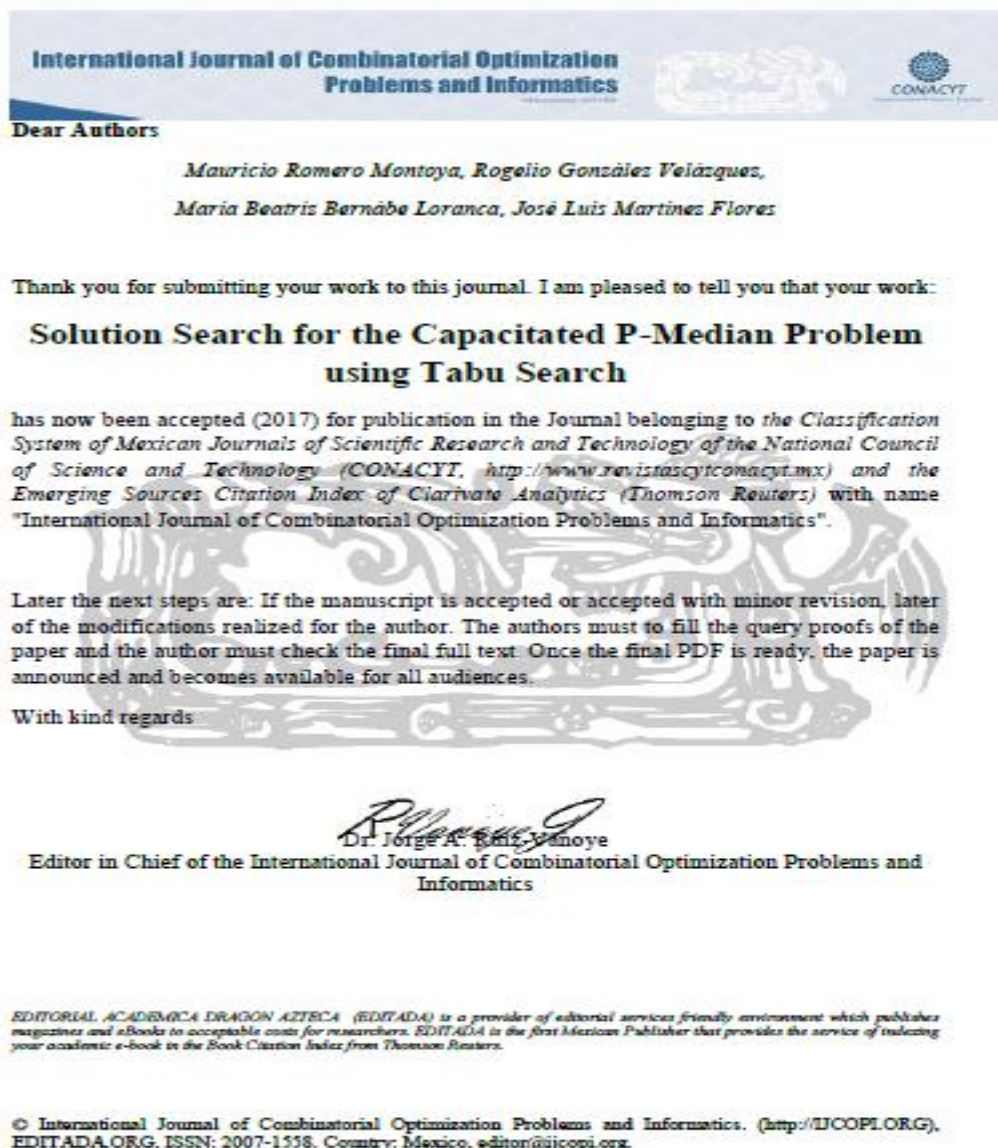


Figure 7 Carta de aceptación de artículo en revista de índice CONACYT